CBCS SCHEME

USN | | | | | | | | | | 20MCA14

First Semester MCA Degree Examination, Feb./Mar. 2022
Mathematical Foundation for Computer Applications

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. For any three sets A, B and C prove that
i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(06 Marks)**

b. A survey of 500 television viewers of sports channel produced the following information. 285 watch football, 195 watch hockey, 115 watch basket ball, 45 watch football and basket ball, 70 watch football and hockey, 50 watch hockey and basketball, 50 do not watch any of the 3 games.
i) How many people in the survey watch all 3 games?
ii) How many people watch exactly one of 3 games?
iii) How many people watch two of 3 games? **(07 Marks)**

c. Explain the Pigeonhole principle. **(07 Marks)**

OR

2 a. Define the following with an example.
i) Cardinality of a set ii) Subset iii) Power set. **(06 Marks)**

b. Find the number of permutations of the digits 1 through 9 in which the blocks 36, 78, 672 do not appear. **(07 Marks)**

c. Find all the eigenvalues and eigen vector corresponding to the largest eigen value of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **(07 Marks)**

Module-2

3 a. Define tautology and prove that
$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology. **(06 Marks)**

b. Test the validity of the argument.
If Ravi goes out with friends, he will not study. If Ravi does not study, his father becomes angry. His father is not angry. $\therefore$ Ravi does not go with friends. **(07 Marks)**

c. Give (i) A direct proof (ii) An indirect proof (iii) A proof by contradiction, for the following statement " If 'n' is an odd integer then n + 9 is an even integer". **(07 Marks)**

OR

4 a. Using laws of logic prove the following :
i) $(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r) \equiv p \wedge q$
ii) $(p \rightarrow q) \wedge (\neg q \wedge (r \wedge \neg q)) \equiv \neg(p \vee q)$ **(06 Marks)**

b. Test the validity of the argument
$\forall x, [p(x) \rightarrow q(x)]$
$\forall x, [q(x) \rightarrow r(x)]$
$\underline{\exists x, \neg r(x)}$
$\therefore \exists x, \neg p(x)$ **(07 Marks)**

c. Write the following proposition in the symbolic form and find its negation. "If all triangles are right angled then no triangle is equiangular. **(07 Marks)**

Module-3

5 a. If $A = \{1, 3, 5\}$, $B = \{2, 3\}$, $C = \{4, 6\}$. Find the following :
i) $(A \cup B) \times C$ ii) $(A \times B) \cup C$ iii) $(A \times B) \cup (B \times C)$ **(06 Marks)**

b. Let $A = \{1, 2, 3, 4\}$ and Let R be the relation on A defined by xRy if and only if "x divided y", written x/y.
   i) Write down R as a set of ordered pairs
   ii) Draw the digraph of R
   iii) Determine the in-degrees and out-degrees of the vertices in the digraph **(07 Marks)**

c. Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4, 4)\}$, be a relation on A. Verify that R is an equivalence relation. **(07 Marks)**

OR

6 a. Let $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4\}$. The relations R and S from A to B are represented by the following matrices. Determine the relations

$\overline{R}$, $R \cup S$, $R \cap S$ an $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$, $M_S = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ **(06 Marks)**

b. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and R be the equivalence relation on A that induces the partition $A = \{1, 2\} \cup \{3\} \cup \{4, 5, 7\} \cup \{6\}$. Find R. **(07 Marks)**

c. Let R be a relation on the set $A = \{1, 2, 3, 4\}$ defined by xRy if and only if x divides y. Prove that (A, R) is a poset. Draw its Hasse diagram. **(07 Marks)**

Module-4

7 a. Find the value of K such that the following distribution represents a finite probability distribution. Hence find its Mean and Variance. Also find $P(x \le 1)$, $P(x > 1)$, $P(-1 < x \le 2)$.

x :	−3	−2	−1	0	1	2	3
P(x) :	K	2K	3K	4K	3K	2K	K

(06 Marks)

b. When a coin is tossed 4 times, find the probability of getting (i) exactly one head (ii) at most 3 heads (iii) atleast 2 heads. **(07 Marks)**

c. The length of telephone conversation in a booth has been an exponential distribution and found an average to be 5 minutes. Find the probability that a random call made from this booth (i) ends less than 5 minutes (ii) between 5 and 10 minutes. **(07 Marks)**

OR

8 a. A random variable x has the following probability density function

$$f(x) = \begin{cases} Kx^2, & -3 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate K and find (i) $P(1 \le x \le 2)$ ii) $P(x \le 2)$ iii) $P(x > 1)$. **(06 Marks)**

b. The probability that a pen manufactured by a factory be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that i) exactly 2 are defective ii) atleast 2 are defective iii) none of them are defective. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75. Given that $A(1) = 0.3413$. **(07 Marks)**

Module-5

9 a. Define the following with an example.
i) Complete graph ii) Bipartite graph iii) Complement of a graph. **(06 Marks)**

b. Show that the following graphs are isomorphic [Refer Fig Q9(b)]

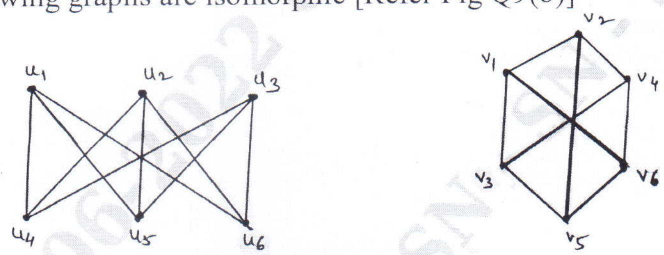

Fig Q9(b) **(07 Marks)**

c. Find the chromatic polynomial and chromatic number for the cycle C_4 [Refer Fig Q9(c)]

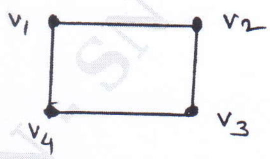

Fig Q9(c) **(07 Marks)**

OR

10 a. Prove that for an undirected graph G = (V, E) the number of vertices of odd degree is even. **(06 Marks)**

b. Explain the Konigsberg Bridge problem. **(07 Marks)**

c. Show that the bipartite graph $K_{2,2}$ and $K_{2,3}$ are planar graphs. [Refer Fig Q10(c)]

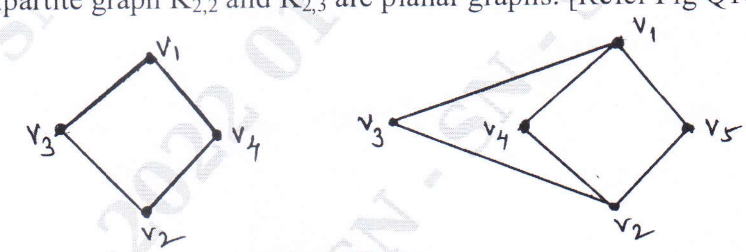

Fig Q10(c) **(07 Marks)**

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